matrix A is a fact that, to a known extent, is random and depends on the elastic moduli E_{ijkl}, the choice of one of the eigenvalues λ_p of the matrix E_{i3k3} and the number n of half-waves in the thickness of the plate.

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HEAT TRANSFER THROUGH A RIGID DISC PRESSED INTO AN ELASTIC HALF-SPACE†

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The axisymmetrical contact problem of the indentation of a rigid disc, modelled by a cylindrical punch, into an elastic half-space is considered. The upper end of the cylinder is subjected to convective heating or cooling and the thermal contact between the punch and the half-space is non-ideal. Outside the region of contact heat exchange occurs with the external medium in accordance with Newton's law. The solution of the thermo-elasticity problem for the half-space is constructed using the Hankel transformation, and the problem of heat conduction for a cylinder is solved by the method of straight lines. The existence of zones where the half-space becomes detached from the punch is established. The temperature fields, heat fluxes and contact stresses in the interacting bodies are found.

1. FORMULATION OF THE PROBLEM

WHEN solving contact problems of thermo-elasticity it is of interest to investigate the phenomenon in which a punch becomes separated from the base [1–3]. However, in these and other

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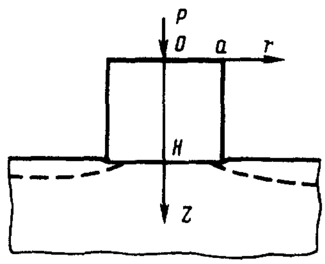

FIG. 1.

investigations, the heat-conduction equation is not solved for the punch, i.e. the temperature of the base of the punch or the heat flux is specified. Below we propose a method of solving the axisymmetrical contact problems of thermo-elasticity when there is incomplete contact between the interacting bodies, taking into account the solution of the heat-conduction problem for a rigid cylindrical punch. Outside the punch we assume that convective heat transfer occurs with the external medium. There is non-ideal thermal contact between the punch and the base.

A rigid cylindrical disc of radius a and height H is pressed with a force P into an elastic half-space (Fig. 1). The upper end of the cylinder is subjected to convective heating or cooling with a heat-transfer coefficient γ_0. Heat exchange occurs between the side surfaces and the external medium in accordance with Newton's law with a heat-transfer coefficient of γ_a. Convective heat exchange occurs with the external medium through the unloaded surface of the half-space with a heat-transfer coefficient γ_h.

To solve this problem we need to integrate the following equations:

$$\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}+\frac{\partial^2 u_r}{\partial z^2}-\frac{u_r}{r^2}+k\frac{\partial\theta}{\partial r}=\frac{\beta}{\mu}\frac{\partial t^{(2)}}{\partial r} \tag{1.1}$$

$$\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}+\frac{\partial^2 u_z}{\partial z^2}+k\frac{\partial\theta}{\partial z}=\frac{\beta}{\mu}\frac{\partial t^{(2)}}{\partial z}$$

$$\frac{\partial^2 t^{(i)}}{\partial z^2}+\frac{1}{r}\frac{\partial t^{(i)}}{\partial r}+\frac{\partial^2 t^{(i)}}{\partial r^2}=0 \quad (i=1,2) \tag{1.2}$$

$$\theta=\frac{u_r}{r}+\frac{\partial u_r}{\partial r}+\frac{\partial u_z}{\partial z}$$

$$k=\frac{\lambda+\mu}{\mu}=\frac{1}{1-2\nu},\quad \beta=(3\lambda+2\mu)\alpha$$

with the temperature boundary conditions

$$z=0:\ \partial t^{(1)}/\partial z=\gamma_0(t^{(1)}-t_c),\quad 0\leqslant r\leqslant a \tag{1.3}$$

$$r=a:\ \partial t^{(1)}/\partial r=-\gamma_a t^{(1)},\quad 0\leqslant z\leqslant H$$

$$z=H:\ \lambda^{(2)}\partial t^{(2)}/\partial z=\lambda^{(1)}\partial t^{(1)}/\partial z,\quad 0\leqslant r\leqslant a \tag{1.4}$$

$$\lambda^{(2)}\partial t^{(2)}/\partial z+\lambda^{(1)}\partial t^{(1)}/\partial z=h(t^{(2)}-t^{(1)}),\quad 0\leqslant r\leqslant a$$

$$\partial t^{(2)}/\partial z=\gamma_h t^{(2)},\quad r>a$$

and the force boundary conditions

$$z=H:\ u_z=f(r),\quad 0\leqslant r\leqslant a;\quad \sigma_z=0,\quad r\geqslant a;\quad \tau_{rz}=0,\quad r<\infty \tag{1.5}$$

where ν is Poisson's ratio, λ and μ are the Lamé coefficients, α is the temperature coefficient of linear expansion, $i=1$ relates to the cylindrical punch and $i=2$ relates to the half-space, γ_0, γ_a, γ_h are the heat-transfer coefficients between the upper end of the cylinder, the side surfaces of the cylinder, the unloaded surface of the half-space on one side and the external medium on the other, respectively, t_c is the temperature of the external medium, h^{-1} is the thermal resistance of the contact, $\lambda^{(1)}$, $\lambda^{(2)}$ are the thermal conductivities of the cylinder and the half-space, and $f(r)$ is the specified value of the deposition of the punch.

2. SOLUTION OF THE PROBLEM FOR THE PUNCH

Using the finite-difference approximation of the heat-conduction equations (1.2) for a cylinder and the boundary conditions (1.3) in the dimensionless radial coordinate $\rho = r/a$, the solution of the problem for a cylinder can be constructed by the method of straight lines [4] in the region

$$G = \{0 \leqslant \rho \leqslant 1,\ 0 \leqslant \zeta \leqslant 1\},\ \zeta = x/M$$

As a result we obtain the following system of linear differential equations:

$$d\mathbf{v}/d\zeta = B\mathbf{v} \tag{2.1}$$

Here

$$\mathbf{v}^T = (t_1^{(1)}, \ldots, t_N^{(1)}, dt_1^{(1)}/d\zeta, \ldots, dt_N^{(1)}/d\zeta)$$

$$B = \begin{Vmatrix} 0 & I \\ B_1 & 0 \end{Vmatrix}$$

$$B_1 =$$

$$= \begin{Vmatrix} \frac{4H^2}{a^2\Delta\rho^2}, -\frac{4H^2}{a^2\Delta\rho^2}, 0, \ldots & & , 0 \\ \vdots & & \\ \underbrace{0, \ldots, 0}_{i-2}, \frac{H^2}{a^2\Delta\rho^2}\left(\frac{1}{2(i-1)} - 1\right), \frac{2H^2}{a^2\Delta\rho^2}, -\frac{H^2}{a^2\Delta\rho^2}\left(1 + \frac{1}{2(i-1)}\right), & & \underbrace{0, \ldots, 0}_{N-i-1} \\ 0, \ldots & , 0, \frac{2H^2}{a^2\Delta\rho^2}, \frac{H^2}{a}\left[\gamma_a\left(1 + \frac{2}{\Delta\rho}\right) + \frac{2}{a\Delta\rho^2}\right] & \end{Vmatrix}$$

where 0 is the zero and I is the unit $N \times N$ matrix $\Delta\rho = \rho_i - \rho_{i-1}$, $i = 1, \ldots, N$ and N is the number of points of subdivision.

The solution of (2.1) can be constructed using the matrix exponential function [5]

$$\mathbf{v}(\zeta) = \exp(B\zeta) \cdot \mathbf{d}$$

where $\mathbf{d}$ is an arbitrary constant vector, found from the boundary conditions (1.3) for each straight line. To calculate the matrix exponential function we use the formula [5]

$$t^{(1)}(\zeta) = \exp(B\zeta) = [\exp(B(\zeta \cdot 2^{-p}))]^{2^p}$$

where $q = \zeta \cdot 2^{-p}$ is chosen in such a way as to ensure that the calculations are stable.

3. SOLUTION OF THE PROBLEM FOR THE HALF-SPACE

Applying the integral Hankel transformation with respect to the coordinate ρ to Eqs (1.1) and (1.2) and the boundary conditions (1.4) and (1.5), the solution of the axisymmetrical equations of thermo-elasticity for an elastic half-space in the Hankel transforms can be represented in the form

$$\bar{t}^{(2)}(\xi,\zeta)=B(\xi)e^{-\xi(\zeta-1)}$$

$$\bar{u}_z(\xi,\zeta)=\xi^2[kC_1(\xi)+2C_2(\xi)+k\xi(\zeta-1)C_2(\xi)]e^{-\xi(\zeta-1)}+$$
$$+{}^1/_2\Lambda\xi^{-1}B(\xi)[1-\xi(\zeta-1)]e^{-\xi(\zeta-1)}$$

$$\bar{\sigma}_z(\xi,\zeta)=-2\mu\xi^3[kC_1(\xi)+C_2(\xi)+k\xi(\zeta-1)C_2(\xi)]e^{-\xi(\zeta-1)}+$$
$$+\mu\xi(\zeta-1)\Lambda B(\xi)e^{-\xi(\zeta-1)}$$

$$\bar{\tau}_{rz}(\xi,\zeta)=2\mu\xi^3[(k-1)C_2(\xi)-kC_1(\xi)-k\xi(\zeta-1)C_2(\xi)]e^{-\xi(\zeta-1)}-$$
$$-\mu\Lambda B(\xi)[1-\xi(\zeta-1)]e^{-\xi(\zeta-1)}$$

$$\rho=\frac{r}{a},\ \zeta=\frac{z}{H},\ \Lambda=\frac{\nu+1}{\nu-1}\alpha=-\frac{3\lambda+2\mu}{\lambda+2\mu}\alpha$$

where $B(\xi)$, $C_1(\xi)$, $C_2(\xi)$ are unknown functions.

By satisfying the last condition of (1.5) and using the relation obtained

$$C_2(\xi)=\frac{k}{k-1}C_1(\xi)+\frac{\Lambda}{2(k-1)\xi^3}B(\xi)$$

the required functions on the area of contact can be represented as follows:

$$\bar{u}_z|_{\zeta=1}=\theta_1\xi^2C_1(\xi)+\theta_3\xi^{-1}B(\xi),\quad \bar{\sigma}_z|_{\zeta=1}=\sigma_1\xi^3C_1(\xi)+\sigma_3B(\xi)$$

$$\bar{t}^{(2)}|_{\zeta=1}=B(\xi),\quad \partial\bar{t}^{(2)}/\partial\zeta|_{\zeta=1}=-\xi HB(\xi)$$

$$\theta_1=k\frac{k+1}{k-1},\quad \theta_3=\frac{\Lambda}{2}\frac{k+1}{k-1},\quad \sigma_1=-\frac{2\mu k^2}{k-1},\quad \sigma_3=-\frac{\mu\Lambda}{k-1}$$

4. SATISFACTION OF THE BOUNDARY CONDITIONS FOR $\xi = 1$

Using the formula for the inversion of the integral Hankel transform and by satisfying the first two force boundary conditions (1.5) we obtain

$$\int_0^\infty \eta\left(\frac{\theta_1}{a^2}\eta^2C_1(\eta)+\frac{a\theta_3}{\eta}C_3(\eta)\right)J_0(\eta\rho)\,d\eta=a^2f(\rho),\quad \rho\leqslant 1 \tag{4.1}$$

$$\frac{1}{a^2}\int_0^\infty \eta\left(\frac{\sigma_1\eta^3}{a^3}C_1(\eta)+\sigma_3C_3(\eta)\right)J_0(\eta\rho)\,d\eta=0,\quad \rho>1 \tag{4.2}$$

where $\eta=\xi a$, while the thermal boundary conditions (1.4) lead to the relations

$$\frac{\lambda^{(1)}}{H}\frac{\partial t^{(1)}}{\partial\zeta}+\frac{\lambda^{(2)}}{a^3}\int_0^\infty \eta^2C_3(\eta)J_0(\eta\rho)\,d\eta=0,\quad \rho\leqslant 1 \tag{4.3}$$

$$\frac{\lambda^{(1)}}{H}\frac{\partial t^{(1)}}{\partial\zeta}+ht^{(1)}=\frac{1}{a^2}\int_0^\infty\left(h+\frac{\lambda^{(2)}\eta}{a}\right)\eta C_3(\eta)J_0(\eta\rho)\,d\eta,\quad \rho\leqslant 1 \tag{4.4}$$

$$\frac{1}{a^2}\int_0^\infty \eta\left(\gamma_h+\frac{\eta}{a}\right)C_3(\eta)J_0(\eta\rho)\,d\eta=0,\quad \rho>1 \tag{4.5}$$

By representing the contact stresses in the form of a series

$$\sigma_z(\rho)=\sum_{n=1}^{N}a_nJ_0(\lambda_n\rho)$$

and introducing the unknown function

$$\chi(\rho) = b_0 + \sum_{n=1}^{N-1} b_n J_0(\lambda_n \rho)$$

into relations (4.2) and (4.5), using the formula for the inversion of the integral Hankel transform, and evaluating certain integrals [6], we obtain relations for $C_1(\eta)$ and $C_3(\eta)$, the substitution of which into (4.1), (4.3) and (4.4) leads to the following equations:

$$\frac{\theta_1}{\sigma_1} a \sum_{k=1}^{N} a_k \lambda_k J_1(\lambda_k) \int_0^\infty \frac{J_0(\eta) J_0(\eta\rho)}{\lambda_k^2 - \eta^2} d\eta +$$

$$+ \left(\theta_3 - \frac{\sigma_3 \theta_1}{\sigma_1}\right) a^2 \left(b_0 \int_0^\infty \frac{J_1(\eta) J_0(\eta\rho)}{\eta(\eta + a\gamma_h)} d\eta + \right.$$

$$\left. + \sum_{k=1}^{N-1} b_k \lambda_k J_1(\lambda_k) \int_0^\infty \frac{1}{\eta + a\gamma_h} \frac{J_0(\eta) J_0(\eta\rho)}{\lambda_k^2 - \eta^2} d\eta \right) = f(\rho), \quad \rho \leqslant 1 \tag{4.6}$$

$$\lambda^{(2)} b_0 \int_0^\infty \frac{\eta}{\eta + a\gamma_h} J_1(\eta) J_0(\eta\rho) d\eta +$$

$$+ \lambda^{(2)} \sum_{k=1}^{N-1} b_k \lambda_k J_1(\lambda_k) \int_0^\infty \frac{\eta^2}{\eta + a\gamma_h} \frac{J_0(\eta) J_0(\eta\rho)}{\lambda_k^2 - \eta^2} d\eta + \frac{\lambda^{(1)}}{H} \frac{\partial t^{(1)}}{\partial \zeta} = 0, \quad \rho \leqslant 1 \tag{4.7}$$

$$b_0 \int_0^\infty \frac{ah + \lambda^{(2)}\eta}{\eta + \alpha\gamma_h} J_1(\eta) J_0(\eta\rho) d\eta +$$

$$+ \sum_{k=1}^{N-1} b_k \lambda_k J_1(\lambda_k) \int_0^\infty \frac{\eta(ah + \lambda^{(2)}\eta)}{\eta + a\gamma_h} \frac{J_0(\eta) J_0(\eta\rho)}{\lambda_k^2 - \eta^2} d\eta = \frac{\lambda^{(1)}}{H} \frac{\partial t^{(1)}}{\partial \zeta} + ht^{(1)}, \quad \rho \leqslant 1 \tag{4.8}$$

where λ_k are the zeros of the Bessel function

$$J_0(\lambda_k) = 0, \quad (k = 1, \ldots, N) \tag{4.9}$$

5. REDUCTION OF THE SOLUTION OF THE PROBLEM TO A SYSTEM OF ALGEBRAIC EQUATIONS

By satisfying relations (4.6)–(4.8) and the boundary conditions (1.3) at a number of equidistant points

$$\rho_i = (i-1)/(N-1), \quad (i = 1, \ldots, N)$$

we obtain a system of $4N$ linear algebraic equations for finding the unknown coefficients a_i, b_i, d_i, ($i = 1, \ldots, N$; $j = 1, \ldots, 2N$), in terms of which we find the required functions

$$t^{(1)}(\rho_i, \zeta) = \sum_{j=1}^{2N} t_{i,j}(\zeta) d_j \tag{5.1}$$

$$q^{(1)}(\rho_i, \zeta) = \frac{\lambda^{(1)}}{a} \frac{\partial t^{(1)}}{\partial \zeta}; \quad (i = 1, \ldots N)$$

$$t^{(2)}(\rho, \zeta) = ab_0 \int_0^\infty \frac{J_1(\eta) J_0(\eta\rho)}{\eta + a\gamma_h} e^{-\eta(\zeta - 1)} d\eta +$$

$$+ a \sum_{n=1}^{N-1} b_n \lambda_n J_1(\lambda_n) \int_0^\infty \frac{\eta}{\eta + a\gamma_h} \frac{J_0(\eta) J_0(\eta\rho)}{\lambda_n^2 - \eta^2} e^{-\eta(\zeta-1)} d\eta$$

$$q^{(2)}(\rho, \zeta) = \frac{\lambda^{(2)}}{a} \frac{\partial t^{(2)}(\rho, \zeta)}{\partial \zeta} = \lambda^{(2)} b_0 \int_0^\infty \frac{\eta J_1(\eta) J_0(\eta\rho)}{\eta + a\gamma_h} e^{-\eta(\zeta-1)} d\eta +$$

$$+ \lambda^{(2)} \sum_{n=1}^{N-1} b_n \lambda_n J_1(\lambda_n) \int_0^\infty \frac{\eta^2}{\eta + a\gamma_h} \frac{J_0(\eta) J_0(\eta\rho)}{\lambda_n^2 - \eta^2} e^{-\eta(\zeta-1)} d\eta$$

$$u_z(\rho, \zeta) = \frac{a}{\sigma_1} \sum_{n=1}^{N} a_n \lambda_n J_1(\lambda_n) \int_0^\infty \left(\theta_1 + \frac{k^2\eta(\zeta-1)}{k-1}\right) \frac{J_0(\eta) J_0(\eta\rho)}{\lambda_n^2 - \eta^2} e^{-\eta(\zeta-1)} d\eta +$$

$$+ a^2 b_0 \int_0^\infty \frac{J_1(\eta) J_0(\eta\rho)}{(\eta + a\gamma_h)\eta} A(\eta, \zeta) e^{-\eta(\zeta-1)} d\eta +$$

$$+ a^2 \sum_{n=1}^{N-1} b_n \lambda_n J_1(\lambda_n) \int_0^\infty \frac{1}{\eta + a\gamma_h} \frac{J_0(\eta) J_0(\eta\rho)}{\lambda_n^2 - \eta^2} A(\eta, \zeta) e^{-\eta(\zeta-1)} d\eta$$

where

$$A(\eta, \zeta) = \theta_3 + \frac{\theta_3}{k+1} \eta(\zeta-1) - \frac{\sigma_3}{\sigma_1}\left(\theta_1 + \frac{k^2\eta(\zeta-1)}{k-1}\right)$$

Computations were carried out for the case when the material of the cylinder was steel $[\lambda^{(1)} = 22\ \text{W/(m K)}]$, and the material of the half-space was aluminium $[\lambda^{(2)} = 209\ \text{W/m K)}]$, $\gamma_0 = \gamma_a = \gamma_h = 10\ \text{m}^{-1}$, $t_c = 473$ K, $h = 10^4\ \text{W/(m}^2\ \text{K)}$, $\alpha = 22.9 \times 10^{-6}\ \text{K}^{-1}$, $a = 1$ m, $H = 0.3$ m, $f = 10^{-4}$ m and $N = 19$.

We obtained the distribution of the contact stresses (Fig. 2). The change in the sign of σ_z in the section $[0, a]$ confirms the existence of zones in which the half-space becomes detached from the cylindrical punch. Consequently, contact between the bodies occurs in the section $[0, \rho_0]$, where $\rho_0 < 1$, and hence the first two conditions of contact (1.4) and the first condition of (1.5) must relate only to $\rho \leq \rho_0$, while the second condition of (1.5) relates to $\rho > \rho_0$. In the section $\rho_0 < \rho < 1$ between the lower end of the cylinder and the external medium convective heat transfer occurs in accordance with Newton's law

$$\zeta = 1:\ \partial t^{(1)}/\partial \zeta = -H\gamma_p t^{(1)}, \quad \rho_0 < \rho < 1 \tag{5.2}$$

where γ_p is the heat-transfer coefficient between the unloaded surface at the lower end of the cylinder and the external medium.

Satisfaction of the changed boundary conditions leads to the need to solve a new system of $4(N-I)$ linear algebraic equations for determining the unknown coefficients. The value of the parameter I is obtained by successive change in the number of points of subdivision in which a change in the sign of the stresses σ_z occurred, i.e. the half-space becomes detached from the punch.

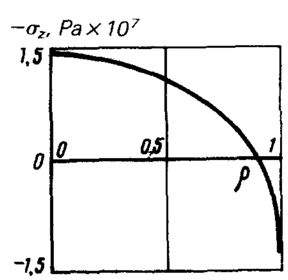

FIG. 2.

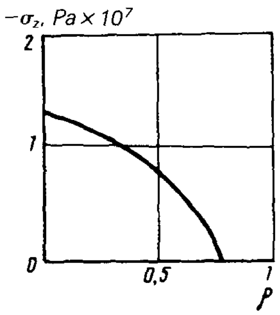

FIG. 3.

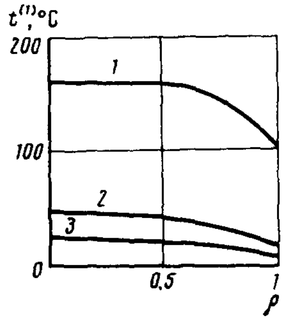

FIG. 4.

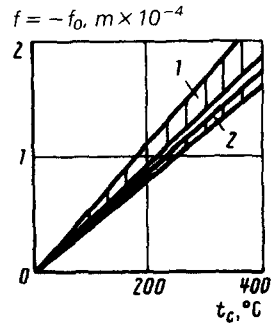

FIG. 5.

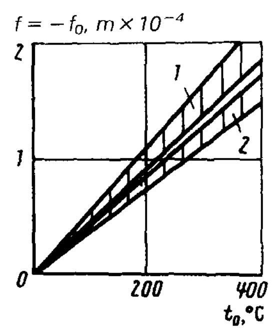

FIG. 6.

The final contact stresses are shown in Fig. 3 for a force P, which was found from the condition of equilibrium of the punch

$$P=2\pi\rho_0^2\sum_{n=1}^{N-1}a_n\lambda_nJ_1(\lambda_n)$$

The temperature and heat fluxes in the contacting bodies and the vertical displacements of the elastic half-space are found from relations (5.1). In Fig. 4 we show, for example, the temperature of the cylinder. Curves 1, 2 and 3 correspond to values of $z = 0$, $z = 0.25$ m and $z = 0.3$ m.

6. CONCLUSIONS

1. The thermal conductivity of the contact h has a considerable effect on the size of the contact zone. It decreases as h increases. [In Fig. 5 region 1 corresponds to $h = 10^4$ W/(m^2 K) while region 2 corresponds to $h = 10^3$ W/(m^2 K)].

2. The heat-transfer coefficient of the upper end of the cylinder with the external medium also has a considerable effect on the size of the contact zone. (In Fig. 6, region 1 corresponds to $\gamma_0 = 10$ m^{-1} while region 2 corresponds to $\gamma_0 = 5$ m^{-1}.)

3. For a given force P a version of contact $\rho_0 \to 0$ is possible (the left boundary of regions 1 and 2 in Figs 5 and 6). No solution of the contact problem exists to the left of the hatched region.

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ON THE THERMO-ELASTICITY PROBLEM OF NON-UNIFORM PLATES†

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The general case of the problem of the thermo-elasticity of non-uniform plates is considered. A formal asymptotic expansion is constructed and the limiting problem (when the thickness of the plate approaches zero) is obtained. The limiting problem in the general case turns out to be different from the classical one, in particular, it contains five unknown functions, and the defining equations contain not only the temperature but also its derivatives (although the material of the plate is assumed to obey the Duhamel–Neumann law). These effects do not occur in uniform plates of constant thickness. This is obviously the reason why the effects stated below have not been mentioned previously, as far as we know.

A GENERAL scheme of the asymptotic method for passage from a three-dimensional problem of the theory of elasticity in a thin region (thickness $\varepsilon \ll 1$) to a problem in the theory of plates was previously proposed in [1]. A case which leads to the classical equations of thermo-elastic plates was considered in [2] (it turns out that it corresponds to the case when the coefficients of thermal expansion of the material of the plate are of the order of ε).

1. FORMULATION OF THE PROBLEM

Suppose a three-dimensional linearly elastic body occupying the region Q_ε of characteristic thickness $\varepsilon \ll 1$ is obtained by repetition of an element P_ε (the periodicity cells, PC) in the x_1x_2 plane (Fig. 1). The condition $\varepsilon \ll 1$ is formalized in the form $\varepsilon \to 0$.

The equations of equilibrium of this body have the form [3]

$$\int_{Q_\varepsilon} \sigma_{ij} v_{i,j}\, d\mathbf{x} + \int_{Q_\varepsilon} \mathbf{f}\mathbf{v}\, d\mathbf{x} = 0 \tag{1.1}$$

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